# The fast and accurate calculation of frequency-dependent response properties using a multiresolution adaptive numerical solver at the basis set limit

## Objectives

We wish to develop a multiresolution numerical solver for the computations of frequencydependent response properties.

- Linear and non-linear response properties
- User-defined precision in molecular property

#### Introduction

We have developed a multiresolution analysis solver to compute linear and non-linear response properties. MRA using multiwavelets provides an adaptive and complete basis, guaranteeing precision in the desired property. Compared to other basisset techniques, MRA also has lower scaling for computing larger molecules. In addition, MRA demonstrated its ability to compute response properties, including CIS excitation energies and static polarizabilities [1, 2]. This work introduces the development of our frequency-dependent solver, which we may use to compute TD-HF/DFT linear and non-linear response properties. We also provide preliminary dynamic polarizabilities calculated in the TDHF formalism compared to standard Gaussian basis sets calculations done in Dalton[3].

#### **Response Properties**

We compute the frequency-dependent polarizability via the calculation of the frequency-dependent response density.

$$\alpha_{xy}(-\omega,\omega) = tr(\hat{v}^{(x)}\hat{\rho}^{(y)}(\omega))$$

The objective of our solver is to calculate the response functions of the response density by solving integral form of the response equations.

$$\begin{aligned} x_p^{(\alpha)}(r) &= -2\hat{G}^{(\alpha)}(k_x) * \left[ \hat{V}^{(0)} x_p^{(\alpha)}(r) + \Gamma_p^{(\alpha)}(r) + V^{(\alpha)} \right] \\ y_p^{(\alpha)}(r) &= -2\hat{G}^{(\alpha)}(k_y) * \left[ \hat{V}^{(0)} y_p^{(\alpha)}(r) + \Gamma_p^{\dagger(\alpha)}(r) + V^{\dagger(\alpha)} \right] \end{aligned}$$

$$k_x = [-2(\epsilon_p + \omega_\alpha)]^{1/2} \quad k_y = [-2(\epsilon_p - \omega_\alpha)]^{1/2}$$

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## MRA Hartree-Fock Polarizability Results



Figure: Average dynamic polarizabilities computed with the MRA multiwavelet basis-set compared to Dunning's correlation-consistent basis sets aug-cc-pVXZ and d-aug-cc-pVXZ. For each polarizability value, the MRA response densities where converged to a 1e-6 difference between iterations. From the data it can be seen that Guassian basis do not all systematically converge to the basis-set limit.

### Conclusion

We successfully validated our MRA solver for the computation of frequency-dependent polarizabilities. We illustrated issues when trying to extrapolate response properties using Gaussian basis-sets as they may not always systematically converge. The extension of this work will be to calculate third-order properties such as hyperpolarizabilities and Raman response from first-order frequency-dependent den-

#### References

[1] Anders Brakestad, Stig Rune Jensen, Peter Wind, Marco D'Alessandro, Luigi Genovese, Kathrin Helen Hopmann, and Luca Frediani.

Static polarizabilities at the basis set limit: A benchmark of 124 species.

16(8):4874-4882.

[2] Jakob S. Kottmann, Sebastian Höfener, and Florian A.

Numerically accurate linear response-properties in the configuration-interaction singles (CIS) approximation. 17(47):31453-31462.

Publisher: The Royal Society of Chemistry.

[3] The dalton quantum chemistry program system. 4(3):269-284.

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