APPH4200 Physics of Fluids

Homework 3: Due Thursday, October 16, 2008.

1. Show that for an axisymmetric, two-dimensional flow with vorticity distribution $\omega = (0, 0, \omega(r))$ in cylindrical polar coordinates (r, ϕ, z) (where 2-dimensional means no motion in the z direction, and no variation of the flow with respect to z), the vorticity equation for an incompressible, constant density fluid in an unbounded domain reduces to

$$\frac{\partial \omega}{\partial t} = \frac{\nu}{r}\frac{\partial}{\partial r}(r\frac{\partial \omega}{\partial r}).$$

   If there is a concentrated line vortex along the z-axis at time t=0 in an otherwise irrotational fluid, show by substitution (that is, you don't have to derive it, just plug it into the equation) that the vorticity distribution at any subsequent time is

$$\omega = \frac{\Phi}{4\pi\nu t}e^{-r^2/4\nu t},$$

   where Φ is a constant proportional to the initial circulation.

   What is the corresponding velocity distribution?

   Explain why viscosity causes the vortex to decay in time even though it was shown in class and in the book that the net viscous force vanishes in a point vortex flow (K&C, pp. 131-134).

2. Kundu & Cohen, Chapter 5, Problem 3.

3. Kundu & Cohen, Chapter 5, Problem 6. Just think of a vortex ring as a dipole of opposite-signed point vortices at a finite distance from one another (*i.e.*, in 2D).

4. Consider the geometry and flow described by Kundu & Cohen, Chapter 4, Problem 11. Imagine that at $t = 0$, the upper plate was at rest, and the fluid had no motion. Describe what happens to the flow if at $t = 0^+$, the upper plate moves with constant velocity, U. Use the Navier-Stokes equation, Eq. 4.45.